

Low-excited charm and charm-strange baryons revisited in the quark-diquark picture

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Assuming a heavy quark-light diquark picture, we systematically study the mass spectra and strong decays of $1P$ and $2S$ charm and charm-strange baryons by the nonrelativistic constituent quark models. Most of the existing charm and charm-strange baryons can be well explained as $1P$ and $2S$ states in the diquark picture. As for the well-determined states, including $\Sigma_c(2455)^{0,+,++}$, $\Sigma_c(2520)^{0,+,++}$, $\Xi'_c(2580)^{0,+}$, $\Xi_c(2645)^{0,+}$, $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Xi_c(2790)^{0,+}$, and $\Xi_c(2815)^{0,+}$, the theoretical results are in good agreement with the experimental data. $\Sigma_c(2800)^{0,+,++}$ can be assigned to a $\Sigma_{c2}(3/2^-)$ or $\Sigma_{c2}(5/2^-)$ state. We prefer to interpret the signal $\Sigma_c(2850)^0$ as a $2S(1/2^+)$ state although the possibility can not be thoroughly excluded at present that this is the same state as $\Sigma_c(2800)^0$. $\Lambda_c(2765)^+$ (or $\Sigma_c(2765)^+$) could be explained as the $\Lambda_c^+(2S)$ state and $\Sigma_{c1}^+(1/2^-)$ state, respectively. We propose to measure the branching ratio of $\mathcal{B}(\Sigma_c(2455)\pi)/\mathcal{B}(\Sigma_c(2520)\pi)$ in future, which may disentangle the puzzle of this state. Our results support $\Xi_c(2980)^{0,+}$ as the first radial excited state of $\Xi_c(2470)^{0,+}$ with $J^P = 1/2^+$. The assignment of $\Xi_c(2930)^0$ is analogous to $\Sigma_c(2800)^{0,+,++}$. Specifically, $\Xi_c(2930)^0$ could be a $1P(3/2^-)$ or $1P(5/2^-)$ Ξ'_c state. Some typical ratios among partial decay widths are provided for experiments to search for these missing states.

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I. INTRODUCTION

At present, the Particle Data Group (PDG) lists 9 charm and 10 charm-strange baryons [1]. They are $\Lambda_c(2286)^+$, $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Lambda_c(2765)^+$ (or $\Sigma_c(2765)^+$), $\Lambda_c(2880)^+$, $\Lambda_c(2940)^+$, $\Sigma_c(2800)^{0,+,++}$, $\Sigma_c(2455)^{0,+,++}$, $\Sigma_c(2520)^{0,+,++}$, $\Xi_c(2470)^{0,+}$, $\Xi'_c(2580)^{0,+}$, $\Xi_c(2645)^{0,+}$, $\Xi_c(2790)^{0,+}$, $\Xi_c(2815)^{0,+}$, $\Xi_c^{(r)}(2930)^0$, $\Xi_c(2980)^{0,+}$, $\Xi_c(3055)^{0,+}$, $\Xi_c(3080)^{0,+}$, and $\Xi_c^{(r)}(3123)^+$. Among them, some new measurements have been performed by experiments in the past several years. The masses and widths of $\Sigma_c(2455)^{0,+,++}$, $\Sigma_c(2520)^{0,+,++}$, $\Lambda_c(2595)^+$, and $\Lambda_c(2625)^+$ have been measured with significantly smaller uncertainties with the efforts of CDF [2] and Belle [3]. Very recently, the Belle Collaboration updated the measurements of $\Xi'_c(2580)^{0,+}$, $\Xi_c(2645)^{0,+}$, $\Xi_c(2790)^{0,+}$, $\Xi_c(2815)^{0,+}$, and $\Xi_c(2980)^{0,+}$ [4]. On the other hand, new decay modes for the higher excited charmed baryon states have been found by experiments. For instance, the decay channel of ΛD^+ was first found for $\Xi_c(3055)^+$ and $\Xi_c(3080)^+$ and the following ratios of branching fractions were first reported by Belle several months ago [5]:

$$\frac{\mathcal{B}(\Xi_c(3055)^+ \rightarrow \Lambda D^+)}{\mathcal{B}(\Xi_c(3055)^+ \rightarrow \Sigma_c^{++} K^-)} = 5.09 \pm 1.01 \pm 0.76,$$

$$\frac{\mathcal{B}(\Xi_c(3080)^+ \rightarrow \Lambda D^+)}{\mathcal{B}(\Xi_c(3080)^+ \rightarrow \Sigma_c^{++} K^-)} = 1.29 \pm 0.30 \pm 0.15,$$

and

$$\frac{\mathcal{B}(\Xi_c(3080)^+ \rightarrow \Sigma_c^{++} K^-)}{\mathcal{B}(\Xi_c(3055)^+ \rightarrow \Sigma_c^{++} K^-)} = 1.07 \pm 0.27 \pm 0.01.$$

Obviously, these new measurements are very useful to understand the nature of these excited charmed baryon states.

Theoretically, the charm and charm-strange baryons¹ which contain one heavy quark and two light quarks occupy a particular position in the baryon physics. Since the chiral symmetry and heavy quark symmetry (HQS) can provide some qualitative insight into the dynamics of charmed baryons, the investigation of charmed baryons could be more helpful for improving our understanding of the mechanism of confinement. The mass spectroscopy and strong decays of charmed baryons have been investigated in various models. So far, the different kinds of quark potential models [6–10], the relativistic flux tube (RFT) model [11, 12], the coupled channel model [13], the QCD sum rule [14–16], and the Regge phenomenology [17] have been applied to study the mass spectra of excited charmed baryons, and so did the Lattice QCD [18, 19]. The strong decay behaviors of charmed baryons have been studied by the heavy hadron chiral perturbation theory (HHChPT) [20, 21], the chiral quark model [22, 23], the 3P_0 model [24], and a non-relativistic quark model [25]. The decays of $1P$ Λ_c and Ξ_c baryons have also been investigated by a light front quark model [26, 27], a relativistic three-quark model [28], and the QCD sum rule [29].

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¹ In the following, we call charm and charm-strange baryons as charmed baryons for brevity.

Although lots of experimental and theoretical efforts have been made at the research of charmed baryons, most of the $1P$ and $2S$ charmed baryons are not yet established. Several candidates, including $\Lambda_c(2765)^+$, $\Sigma_c(2800)^{0,+,++}$, $\Xi_c(2930)^0$, and $\Xi_c(2980)^{0,+}$ are still in controversy. $\Lambda_c(2765)^+$ was first observed by CLEO Collaboration in the decay channel of $\Lambda_c(2765)^+ \rightarrow \Lambda_c^+ \pi^+ \pi^-$ [30], and then was confirmed by Belle in the mode $\Sigma_c(2455)\pi$ [31, 32]. Because both Λ_c and Σ_c excitations can decay through $\Lambda_c^+ \pi^+ \pi^-$ and $\Sigma_c(2455)\pi$, we even do not know whether $\Lambda_c(2765)^+$ is the Λ_c or Σ_c state, or their overlapping structure [33]. In the e^+e^- annihilation process, an isotriplet state, $\Sigma_c(2800)^{0,+,++}$, was observed by Belle in the channel of $\Lambda_c^+ \pi$, and tentatively identified as the Σ_{c2} state with $J^P = 3/2^-$ [34]. Interestingly, another neutral resonance was later found by BaBar in the process of $B^- \rightarrow \Sigma_c^{*0} \bar{p} \rightarrow \Lambda_c^+ \pi^- \bar{p}$ with the mass $2846 \pm 8 \pm 10$ MeV and decay width 86^{+33}_{-22} MeV [35]. The higher mass and the weak evidence of $J = 1/2$ indicate that the signal observed by BaBar might be different from the Belle's observation. In this paper, we will denote the signal discovered by BaBar as $\Sigma_c(2850)^0$. $\Xi_c(2930)^0$ was only seen by BaBar in the decay mode $\Lambda_c^+ K^-$ [36], and it still needs more confirmation. $\Xi_c(2980)^{0,+}$ was first reported by Belle in the channels $\Lambda_c^+ K^- \pi^+$ and $\Lambda_c^+ K_S^0 \pi^-$ [37], and was later confirmed by Belle [4, 38] and BaBar [39] in the channels $\Xi_c'(2580)\pi$, $\Xi_c(2645)\pi$ and $\Sigma_c(2455)K$, respectively. However, the decay widths reported by Refs. [4, 37–39] were quite different² from each other. Obviously, a systematic study of masses and decays is required for these unestablished charmed baryons. More importantly, most of $2S$ and $1P$ charmed baryons have not yet been detected by any experiments. Such a research can also help the future experiments find them.

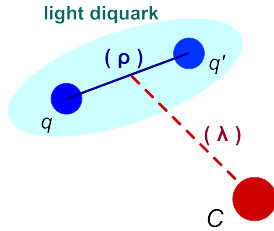


FIG. 1: Illustration of a charmed baryon resonance in the heavy quark-light diquark picture. The symbol of q and q' represents either u , or d , or s quark, and c refers to a charm quark.

To study the dynamics of baryons, one crucial question which should be answered is “What are the relevant degrees of freedom in a baryon?” [44]. In a constituent quark model, a baryon system consists of three confined quarks. Then the dynamics of a baryon resonance is surely more complex than a meson. Due to the HQS, however, the dynamics of charmed baryons could be greatly simplified. The HQS sug-

gests that the couplings between c quark and two light quarks are weak [45]. Therefore, two light quarks in a charmed baryon first couple with each other to form a light diquark, then the diquark couples with a charm quark, and finally a charmed baryon resonance is formed. Accordingly, the dynamics of three quarks in the baryon system can be separated into two parts by the Jacobi coordinates in the following way. The effective degrees of freedom between two light quarks are denoted as the ρ mode while the effective degrees of freedom between the light diquark and the c quark as the λ mode (see Fig. 1). Both ρ and λ modes can in principle be excited as a baryon state. Due to the heavier mass of a c quark, the excitation energies of the ρ and λ modes are different in the charmed baryons. For the ordinary confining potential, such as the linear or harmonic form, the excited energy of the ρ mode is larger than the λ mode [46]. Hence the low excited charmed baryons may be dominated by the λ mode excitations. Recently, the investigation by Yoshida *et al.* confirmed this point [47]. Furthermore, they find that the ρ and λ modes are well separated for the charmed and bottom baryons, which means the component of the ρ mode can be ignored for the low excited charmed baryons. Interestingly, the works [9–11] have also shown that the masses of existing charmed baryons can be explained by the λ mode. So a study of strong decays of the low excited charmed baryons is an important complement to these works [9–11, 47].

In the present work, both the mass spectra and strong decays will be calculated for the $1P$ and $2S$ charmed baryon states. Here the heavy quark-light diquark picture of Refs. [9–11] will be used. The concept of diquark in these works should be understood in the narrow sense that the effective degrees of freedom within the light diquark is frozen. In other words, only the states with lower excited energies will be studied in our paper. Fortunately, most of the observed charmed baryons can be appropriately assigned by our results. The paper is arranged as follows. In Sec. II, the mass spectra of low-excited charmed baryons are calculated by the nonrelativistic quark potential model. In Sec. III, the strong decays are investigated by the Eichten-Hill-Quigg (EHQ) decay formula. Finally, the paper ends with the conclusion and outlook. Some detailed calculations and definitions are collected in the Appendixes.

II. MASS SPECTRA

Generally the light diquark in the baryons can be classified into two kinds: one is flavor symmetric and another is flavor antisymmetric. Constrained by the Pauli's exclusion principle, the total wave function of the light diquark should be antisymmetric in exchange of two quarks. Since the spatial and color parts of a diquark are always symmetric and antisymmetric, respectively, the function, $|\text{flavor}\rangle \times |\text{spin}\rangle$, should be symmetric. Therefore, the scalar diquark $[qq]$ ($S = 0$) is always flavor antisymmetric, and the axial-vector diquark $\{qq\}$ ($S = 1$) is flavor symmetric. In terms of the Jaffe's terminology, the “scalar” and “vector” diquarks are named as the “good” and “bad” diquarks [48]. The color-spin interaction between two light quarks gives the good diquark in the

² More experimental information about the charmed baryons can be found in Refs. [40–43].

Λ_c^+ baryons the mass about 200 MeV lighter than the bad diquark in the $\Sigma_c^{0,+,++}$ baryons. By considering the flavor SU(3) symmetry, the diquark formed by s and q ($q=u$ or d quark) in the charm-strange baryons can also be classified in the similar way. Henceforth, we will call the Λ_c^+ and $\Xi_c^{0,+}$ baryons the \mathcal{G} -type baryons, and $\Sigma_c^{0,+,++}$ and $\Xi_c^{0,+}$ the \mathcal{B} -type baryons for convenience.

In the heavy quark-light diquark picture, a color singlet baryon system should be formed as $3_q \otimes 3_{q'} \otimes 3_Q = \bar{3}_{di} \otimes 3_Q = \bar{1}_{Q-di}$ (see Fig. 1). The abbreviation “ di ” here denotes the diquark system. Since the degrees of freedom of two light quarks are frozen, the light diquark can be treated as a block with the antitriplet color structure and peculiar size. In this way, a heavy baryon could be treated as a quasi two-body system. If this is true, the similarity of dynamics should exist between baryons with a single heavy flavor and heavy-light mesons [49]. More importantly, the scenario of a heavy quark-light diquark picture is not contradictory to the HQS. If the color-magnetic interaction is the principal effects of quark correlations, two light quarks in the heavy-baryon system are expected to strongly correlate since the color-spin interaction is proportional to the inverse of the quark masses. Thus, they may develop into a diquark [48]³. In this way, the dynamics of a heavy quark in a heavy baryon should finally couple to the light degrees of freedom, *i.e.*, diquark.

Because of similarity of the dynamics, the quark potential models which have been applied to calculate the meson spectra may be extended to investigate the low excited charmed baryons. Here, a nonrelativistic quark potential model is employed to calculate the mass spectra of excited charmed baryons. In this model, the Hamiltonian is written as

$$H = \sum_{i=1}^2 \left(m_i + \frac{p_i^2}{2m_i} \right) + H_{Q-di}^{conf} + \underbrace{H_{Q-di}^{cont} + H_{Q-di}^{ten} + H_{Q-di}^{SO}}_{H_s}, \quad (1)$$

The last three terms represent the spin-dependent forces which are taken from Ref. [52]. The H_{Q-di}^{conf} term contains a Coulomb term from one gluon exchange approximation at short distances and a phenomenological confining term, which is written as [53]

$$H_{Q-di}^{conf} = -\frac{4}{3} \frac{\alpha_s}{r} + br^v - C, \quad (2)$$

where the constant C is the renormalization constant which is used to fit the total energy to the physical mass spectrum. The static potential of Eq. (2) is slightly different from the Cornell potential which has the form $-a/r + br + C$ [54]. The color contact interaction is usually given by the following form

$$H_{Q-di}^{cont} = \frac{32\pi}{9} \frac{\alpha_s}{m_Q m_{di}} \tilde{\delta}_\sigma(r) \vec{S}_Q \cdot \vec{S}_{di}, \quad (3)$$

where \vec{S}_Q and \vec{S}_{di} refer to the heavy quark and diquark spins. A Gaussian-smeared function $(\sigma/\sqrt{\pi})^3 e^{-\sigma^2 r^2}$ is normally used for $\tilde{\delta}_\sigma(r)$ [55]. If the SU(3) flavor symmetry is kept intact for the charmed baryons, we may modify the color contact interaction as

$$H_{Q-di}^{cont} = \frac{32}{9} \frac{\alpha_s \sigma^3}{\sqrt{\pi} m_Q} e^{-\sigma^2 r^2} \vec{S}_Q \cdot \vec{S}_{di}, \quad (4)$$

where the mass of light diquark is just assumed as a unit. This assumption is supported by the mass differences of the 1S \mathcal{B} -type charmed baryons,

$$\begin{aligned} \Sigma_c(2520)^{++} - \Sigma_c(2455)^{++} &= 64.44^{+0.25}_{-0.24} \text{ MeV}; \\ \Xi_c(2645)^+ - \Xi_c'(2580)^+ &= 70.2 \pm 3.0 \text{ MeV}; \\ \Omega_c(2770)^0 - \Omega_c(2695)^0 &= 70.7 \pm 2.6 \text{ MeV}. \end{aligned}$$

The mass differences shown above are mainly due to the color contact interaction in the quark potential model. Clearly, these values are almost independent of the diquark masses. The color tensor interaction is

$$H_{Q-di}^{ten} = \frac{4}{3} \frac{\alpha_s}{m_Q m_{di}} \frac{1}{r^3} \left(\frac{3(\vec{S}_Q \cdot \vec{r})(\vec{S}_{di} \cdot \vec{r})}{r^2} - \vec{S}_Q \cdot \vec{S}_{di} \right), \quad (5)$$

Finally, H_{Q-di}^{SO} denotes the spin-orbit interaction which contains two terms. One is the color magnetic interaction which arises from one-gluon exchange

$$H_{Q-di}^{SO(cm)} = \frac{4}{3} \frac{\alpha_s}{r^3} \left(\frac{\vec{S} \cdot \vec{L}}{m_Q m_{di}} + \frac{\vec{S}_Q \cdot \vec{L}}{m_Q^2} + \frac{\vec{S}_{di} \cdot \vec{L}}{m_{di}^2} \right), \quad (6)$$

where \vec{S} denotes the spin of a baryon, $\vec{S} = \vec{S}_Q + \vec{S}_{di}$. Another spin-orbit interaction is the Thomas-precession term

$$H_{Q-di}^{SO(tp)} = -\frac{1}{2r} \frac{\partial H_{Q-di}^{conf}}{\partial r} \left(\frac{\vec{S}_Q \cdot \vec{L}}{m_Q^2} + \frac{\vec{S}_{di} \cdot \vec{L}}{m_{di}^2} \right), \quad (7)$$

To reflect the importance of the heavy quark symmetry, we rewrite the spin-dependent interactions as

$$H_s = V_{ss} \vec{S}_Q \cdot \vec{S}_{di} + V_1 \vec{S}_{di} \cdot \vec{L} + V_2 \vec{S}_Q \cdot \vec{J}_{di} + V_t \hat{S}_{12}. \quad (8)$$

The degrees of freedom of the diquark are characterized by its total angular momentum \vec{J}_{di} , *i.e.*, $\vec{J}_{di} = \vec{S}_{di} + \vec{L}$. Obviously, the orbital angular momentum \vec{L} of a charmed baryon in the diquark picture is defined by the angular momentum between diquark and c quark, *i.e.*, $\vec{L} = \vec{L}_\lambda$. The tensor operator is defined as $\hat{S}_{12} = 3(\vec{S}_Q \cdot \vec{r})(\vec{S}_{di} \cdot \vec{r})/r^2 - \vec{S}_Q \cdot \vec{S}_{di}$.

With the confining term of Eq. (2), the coefficients V_{ss} , V_1 ,

³ In fact, the existence of diquark correlations was partly confirmed by lattice QCD [50] and the Bethe-Salpeter equation [51].

V_1 , and V_t in Eq. (8) are defined as

$$\begin{aligned} V_{ss} &= \frac{1}{m_Q} \left[\frac{32\alpha_s}{9\sqrt{\pi}} \sigma^3 e^{-\sigma^2 r^2} - \frac{1}{m_Q} \left(\frac{2\alpha_s}{3r^3} - \frac{b\nu}{2} r^{\nu-2} \right) - \frac{4\alpha_s}{3r^3} \frac{1}{m_{di}} \right]; \\ V_1 &= \frac{1}{m_{di}} \left[\frac{1}{m_{di}} \left(\frac{2\alpha_s}{3r^3} - \frac{b\nu}{2} r^{\nu-2} \right) + \frac{4\alpha_s}{3r^3} \frac{1}{m_Q} \right]; \\ V_2 &= \frac{1}{m_Q} \left[\frac{1}{m_Q} \left(\frac{2\alpha_s}{3r^3} - \frac{b\nu}{2} r^{\nu-2} \right) + \frac{4\alpha_s}{3r^3} \frac{1}{m_{di}} \right]; \\ V_t &= \frac{4\alpha_s}{3r^3} \frac{1}{m_Q m_{di}}. \end{aligned} \quad (9)$$

In our calculation, the following Schrödinger equation is solved for the s -wave states:

$$\left[\sum_{i=1}^2 \left(m_i + \frac{p_i^2}{m_i} \right) + H_{Q-di}^{conf} + H_{Q-di}^{cont} \right] \Psi = E \Psi. \quad (10)$$

For the orbital excitations, all spin-dependent interactions are treated as the leading-order perturbations. Our calculation indicates that the color contact interaction can be ignored for the orbital excitations.

Two bases are considered for convenience to extract the mass matrix elements. One is the eigenstates $|S_{di}, L, j_{di}, S_Q, J\rangle$ ($j-j$ coupling) and another is $|S_{di}, S_Q, S, L, J\rangle$ ($L-S$ coupling). The relationship between these two bases is given by

$$\begin{aligned} |[S_{di}, L]_{j_{di}}, [S_Q]_J\rangle &= \sum_S (-1)^{S_{di}+S_Q+L+J} \sqrt{(2J_{di}+1)(2S+1)} \\ &\times \begin{Bmatrix} S_{di} & L & j_{di} \\ J & S_Q & S \end{Bmatrix} |[S_{di}, S_Q]_S, [L]_J\rangle, \end{aligned} \quad (11)$$

Due to $S_{di} = 0$ for a good diquark, only $V_2 \vec{S}_Q \cdot \vec{j}_{di}$ survives for the Λ_c^+ and $\Xi_c^{0,+}$ baryons. With a bad diquark, however, $\Sigma_c^{0,+,++}$, $\Xi_c'^{0,+}$ and Ω_c^0 have more complicated splitting structures. Within the framework of the heavy quark effective theory, the spin of an axial-vector diquark, j_{di} , first couples with the orbital angular momentum L . As illustrated in Fig. (2), in the heavy quark limit $m_c \rightarrow \infty$, there are only three states which are characterized by j_{di} for $1P$ charmed baryons. When the heavy quark spin S_Q couples with j_{di} , the degeneracy is resolved and the five states appear. They are two $J' = 1/2^-$, two $J' = 3/2^-$, and one $J' = 5/2^-$ states. Lastly, the states with the same J^P mix with each other by the interactions of $V_{ss} \vec{S}_Q \cdot \vec{S}_{di}$ and $V_t \hat{S}_{12}$, and the physical states are formed.

For $1P$ states with $J^P = 1/2^-$, the mass matrix is given by

$$\langle \Phi_{1/2} | H_S | \Phi_{1/2} \rangle = \begin{pmatrix} -2V_1 - 4V_t & \frac{V_{ss}-4V_t}{\sqrt{2}} \\ \frac{V_{ss}-4V_t}{\sqrt{2}} & -V_1 - V_2 - \frac{V_{ss}}{\sqrt{2}} - 2V_t \end{pmatrix},$$

in the $|S_{di}, L, j_{di}, S_Q, J\rangle$ basis which, in the following, is represented as $|j_{di}, J^P\rangle$ for brevity. Similarly, for two states with $J^P = 3/2^-$,

$$\begin{aligned} \langle \Phi_{3/2} | H_S | \Phi_{3/2} \rangle &= \begin{pmatrix} -V_1 + \frac{V_2}{2} + \frac{V_{ss}}{4} + 4V_t & \frac{V_{ss}-4V_t}{\sqrt{2}} \\ \frac{V_{ss}-4V_t}{\sqrt{2}} & V_1 - \frac{3V_2}{2} - \frac{3V_{ss}}{4} + \frac{4V_t}{5} \end{pmatrix}. \end{aligned}$$

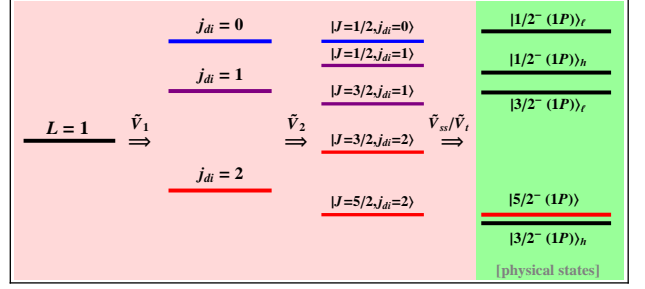


FIG. 2: Schematic diagram for the splittings of p -wave Σ_c and Ξ'_c . Here $\vec{j}_l = \vec{L} + \vec{j}_{di}$ and subindices ℓ and h of the last column mean low and high states in mass after including \tilde{V}_{ss} and \tilde{V}_t interactions.

For the $J^P = 5/2^-$ state,

$$\begin{aligned} \langle j_l = 2, J^P = 5/2^- | H_S | j_l = 2, J^P = 5/2^- \rangle \\ = V_1 + V_2 + \frac{1}{2} V_{ss} - \frac{6}{5} V_t. \end{aligned}$$

The notations $|\Phi_{1/2}\rangle$ and $|\Phi_{3/2}\rangle$ appearing above are defined by

$$|\Phi_J\rangle = \begin{pmatrix} |j_l = J - 1/2, J^P\rangle \\ |j_l = J + 1/2, J^P\rangle \end{pmatrix}.$$

The mass matrix of $1D$ states can also be obtained by the similar procedure. As shown above, there are seven parameters in the nonrelativistic quark potential model. They are m_Q , m_{di} , b , α , γ , ν , and $C_{qq'}$. All values of parameters are listed in Table I. If the SU(3) flavor symmetry is taken into account for the charm and charm-strange baryons, the dynamics of Λ_c^+ states should be like Ξ_c . The case of Σ_c and Ξ'_c is alike. Thereby, we select the same value of γ for Λ_c^+ and Ξ_c , so do the case of Σ_c and Ξ'_c .

TABLE I: Values of the parameters of the nonrelativistic quark potential model. The unit of b is $\text{GeV}^{\nu+1}$ which varies depending on each value of ν .

m_c	1.68 GeV	b	0.145	C_{Λ_c}	0.233 GeV
$m_{[qq]}$	0.45 GeV	α	0.45	C_{Σ_c}	0.100 GeV
$m_{[qs]}$	0.63 GeV	$\nu_{[\Lambda_c, \Xi_c]}$	0.84	C_{Ξ_c}	0.156 GeV
$m_{[qq]}$	0.66 GeV	$\nu_{[\Sigma_c, \Xi'_c]}$	0.70	$C_{\Xi'_c}$	0.060 GeV
$m_{[qs]}$	0.78 GeV	σ	1.00 GeV		

The masses of the good diquarks $[qq]$ and $[qs]$ have been fixed as 450 MeV and 630 MeV by the RFT model [11]. The bad diquark masses may be evaluated by the following relationships

$$\begin{aligned} \frac{4 \times \Sigma_c(2520) + 2 \times \Sigma_c(2455)}{6} - \Lambda_c(2286) &\approx 210 \text{ MeV} \\ \frac{4 \times \Xi_c^*(2645) + 2 \times \Xi'_c(2580)}{6} - \Xi_c(2470) &\approx 150 \text{ MeV}. \end{aligned}$$

Evidently, the bad diquark $\{qq\}$ is about 210 MeV heavier than the good diquark $[qq]$. The bad diquark $\{qs\}$ is about 150 MeV heavier than the good diquark $[qs]$. We have taken the typical values in the quark potential models for m_c , b , α , and ν (see in Table I). It is an effective method to investigate charmed baryons in a diquark picture. We do not expect the value of ν to be the same both for \mathcal{G} -type and \mathcal{B} -type baryons. Here, ν of Λ_c^+/Ξ_c is slightly larger than Σ_c/Ξ'_c . The predicted masses of low excited charmed baryons are collected in Tables II and III.

As mentioned earlier, the nonzero off-diagonal elements in mass matrices of $\langle \Phi_{1/2} | H_S | \Phi_{1/2} \rangle$ and $\langle \Phi_{3/2} | H_S | \Phi_{3/2} \rangle$ cause the mixing between two states with the same J^P but different j_{di} . However, the mechanism of mixing effects in hadron physics is still unclear. In principle, a physical hadron state with a specific J^P comprise all possible Fock states with the same total spin and parity. As the most famous member of the XYZ family, $X(3872)$ may be explained as a mixture between charmonium and molecular state with $J^{PC} = 1^{++}$ [56]. Here we take the $|j_{di}, J^P\rangle$ basis to describe the mixing for the \mathcal{B} -type baryons. Then two physical states characterized by different masses can be denoted as

$$\begin{pmatrix} |High, J^P\rangle \\ |Low, J^P\rangle \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} |J - 1/2, J^P\rangle \\ |J + 1/2, J^P\rangle \end{pmatrix}. \quad (12)$$

For example, two $1P \Sigma_c$ states with $J^P = 1/2^-$ can be represented as

$$\begin{pmatrix} \Sigma_c(2765) \\ \Sigma_c(2702) \end{pmatrix} = \begin{pmatrix} \cos 125.4^\circ & \sin 125.4^\circ \\ -\sin 125.4^\circ & \cos 125.4^\circ \end{pmatrix} \begin{pmatrix} |0, 1/2^-\rangle \\ |1, 1/2^-\rangle \end{pmatrix}. \quad (13)$$

Here we have denoted the physical states by their masses (see Table II). The mixing angles for other states in Tables II and III with the same J^P are listed in Table IV.

Our results of mixing angles in Table IV indicate that the heavier $1/2^-$ state, $\Sigma_c(2765)$, has a dominant $|1, 1/2^-\rangle$ component, while $\Sigma_c(2702)$ does a dominant $|0, 1/2^-\rangle$ component. For two $3/2^-$ states, the light $\Sigma_c(2785)$ has dominant $|2, 1/2^-\rangle$ component, while the heavy $\Sigma_c(2798)$ $|1, 1/2^-\rangle$ component. The mixing of $2P$ states is similar to the $1P$ states. For the $1D$ states, one notices that both $3/2^+$ and $5/2^+$ with the heavier masses have a dominant component of smaller j_{di} .

The uncertainty may exist for the mixing angles. Firstly, the loop corrections to the spin-dependent one-gluon-exchange potential may be important for the heavy-light hadrons. As an example, the low mass of $D_s(2317)^\pm$ with respect to the expectations [52] can be well explained by the corrected spin-dependent potential [60, 61]. If we use this potential in our calculation, of course, the mixing angle will change. Secondly, the mixing angles depend on the parameters. Thirdly, there are other mechanisms, *e.g.*, hadron loop effects [62], which may contribute to the mixing phenomenon in hadron physics. Anyway, we expect that the mixing angles in Table IV reflect main features of the mixing states. Due to the uncertainties of the mixing angles, we ignore the mixing effects as the first step to study the decays of charmed excitations in the next Subsection. Obviously, it is a good approximation only when the mixing effects are not large. Luckily, this crude

procedure is partially supported by the analysis of charmed mesons [63–65]. If the decay properties obtained in this way describe principal characteristics of the mixing states, the angles obtained by the potential model may be overestimated. In the next Section, we will also discuss the mixing effects for the decays of the relevant states.

In the next Section, the two-body strong decays will be calculated for the $1P$ and $2S$ charmed baryons where the simple harmonic oscillator (SHO) wave functions are used to evaluate the transition factors via the 3P_0 model. Following the method of Ref. [66], all values of the SHO wave function scale, denoted as β in the following, are calculated (see Table V). The values of β reflect the distances between the diquark and c quark.

In our calculation of strong decays, we will consider the structures of light diquarks. What is more is that the possible final states of an excited charmed baryon may contain a light flavor meson, a charmed meson, a light flavor baryon, *e.g.*, π , K , D , p , and Λ . For the β of these hadrons, the following potential will be used

$$V(r) = \mathbf{F}_{q_1} \cdot \mathbf{F}_{q_2} \left(\frac{\alpha_s}{r} - \frac{3}{4}br + \frac{3}{4}C \right) + \frac{32\alpha_s\sigma^3 e^{-\sigma^2 r^2}}{9\sqrt{\pi}m_q m_q} \vec{S}_{q_1} \cdot \vec{S}_{q_2}, \quad (14)$$

where

$$\langle \mathbf{F}_{q_1} \cdot \mathbf{F}_{q_2} \rangle = \begin{cases} -\frac{4}{3} & \text{for } q_1 \bar{q}_2 \\ -\frac{2}{3} & \text{for } q_1 q_2 \end{cases} \quad (15a)$$

$$\quad (15b)$$

Here, the parameters α_s and b are also given by 0.45 and 0.145 GeV as in Table I, respectively. To reproduce the masses of light diquarks in Table I, the masses of u/d , s are fixed as 0.195 GeV and 0.380 GeV. While σ and C are treated as adjustable parameters, the masses of π/ρ , K/K^* , D/D^* , p/Δ , and Λ families are fitted with experimental data. Meanwhile, the values of β for the corresponding states are also obtained, which are collected in Table VI.

III. STRONG DECAYS

In this section, we will use the formula provided by Eichten, Hill, and Quigg (EHQ) [67] to extract the decay widths of excited charmed baryons. Since the dynamical behavior of the heavy-light hadrons is governed by the light degrees of freedom in the limit of heavy quark symmetry, a doublet formed by two states with the same j_{di} but different J shall have the similar decay properties. More specifically, the transitions between two doublets should be determined by a single amplitude which is proportional to the products of four Clebsch-Gordan coefficients [45]. Some typical ratios of excited charmed baryons with negative-parity were predicted by this law [45]. Later, a more concise formula (the EHQ formula) was proposed for the widths of heavy-light mesons [67]. The EHQ formula has been applied systematically to the decays of excited open-charm mesons [63–65]. Recently, the

TABLE II: Predicted masses for Λ_c^+ and Ξ_c states of ours and other approaches in Refs. [10, 11, 57, 58] compared to experimental data [1] (in MeV).

States	Λ_c^+ baryons					Ξ_c baryons				
	PDG [1]	Prediction	Ref. [10]	Ref. [11]	Ref. [57]	PDG [1]	Prediction	Ref. [10]	Ref. [11]	Ref. [58]
$ 1S, 1/2^+\rangle$	2286.86	2286	2286	2286	2265	2470.88	2470	2476	2467	2466
$ 2S, 1/2^+\rangle$	2766.6	2772	2769	2766	2775	2968.0	2940	2959	2959	2924
$ 3S, 1/2^+\rangle$		3116	3130	3112	3170		3265	3323	3325	
$ 1P, 1/2^-\rangle$	2592.3	2614	2598	2591	2630	2791.8	2793	2792	2779	2773
$ 1P, 3/2^-\rangle$	2628.1	2639	2627	2629	2640	2819.6	2820	2819	2814	2783
$ 1D, 3/2^+\rangle$		2843	2874	2857	2910	3054.2	3033	3059	3055	3012
$ 1D, 5/2^+\rangle$	2881.53	2851	2880	2879	2910	3079.9	3040	3076	3076	3004
$ 2P, 1/2^-\rangle$	2939.3	2980	2983	2989	3030	3122.9	3140	3179	3195	
$ 2P, 3/2^-\rangle$		3004	3005	3000	3035		3164	3201	3204	

TABLE III: Predicted masses for Σ_c and Ξ_c' states of ours and other approaches in Refs. [9, 10, 57, 59] compared to experimental data [1] (in MeV).

States	Σ_c baryons						Ξ_c' baryons			
	PDG [1]	Prediction	Ref. [9]	Ref. [10]	Ref. [57]	Ref. [59]	PDG [1]	Prediction	Ref. [9]	Ref. [10]
$ 1S, 1/2^+\rangle$	2452.9	2456	2439	2443	2440	2452	2575.6	2579	2579	2579
$ 1S, 3/2^+\rangle$	2517.5	2515	2518	2519	2495	2501	2645.9	2649	2654	2649
$ 2S, 1/2^+\rangle$	2846 ^a	2850	2864	2901	2890	2961		2977	2984	2983
$ 2S, 3/2^+\rangle$		2876	2912	2936	2985	2996		3007	3035	3026
$ 3S, 1/2^+\rangle$		3091		3271	3035	3381		3215		3323
$ 3S, 3/2^+\rangle$		3109		3293	3200	3403		3236		3396
$ 1P, 1/2^-\rangle_l$		2702	2795	2713	2765	2832		2839	2928	2854
$ 1P, 1/2^-\rangle_h$	2766.6	2765	2805	2799	2770	2841		2900	2934	2936
$ 1P, 3/2^-\rangle_l$		2785	2761	2773	2770	2812	2931	2921	2900	2912
$ 1P, 3/2^-\rangle_h$	2801	2798	2798	2798	2805	2822		2932	2931	2935
$ 1P, 5/2^-\rangle$		2790	2799	2789	2815	2796		2927	2921	2929
$ 1D, 1/2^+\rangle$		2949	3014	3041	3005			3075	3132	3163
$ 1D, 3/2^+\rangle_l$		2952	3005	3040	3060			3089	3127	3160
$ 1D, 3/2^+\rangle_h$		2964	3010	3043	3065			3081	3131	3167
$ 1D, 5/2^+\rangle_l$		2942	2960	3023	3065			3091	3087	3153
$ 1D, 5/2^+\rangle_h$		2962	3001	3038	3080			3077	3123	3166
$ 1D, 7/2^+\rangle$		2943	3015	3013	3090			3078	3136	3147
$ 2P, 1/2^-\rangle_l$		2971	3176	3125	3185	3245		3094	3294	3267
$ 2P, 1/2^-\rangle_h$		3018	3186	3172	3195	3256		3144	3300	3313
$ 2P, 3/2^-\rangle_l$		3036	3147	3151	3195	3223		3172	3269	3293
$ 2P, 3/2^-\rangle_h$		3044	3180	3172	3210	3233		3165	3296	3311
$ 2P, 5/2^-\rangle$		3040	3167	3161	3220	3203		3170	3282	3303

^a The mass value for the $|2S, 1/2^+\rangle$ state is taken from the measurement of BaBar [35].

TABLE IV: The mixing angles for the $1P$, $2P$, and $1D$ Σ_c/Ξ_c' states.

	$1P(1/2^-)$	$1P(3/2^-)$	$2P(1/2^-)$	$2P(3/2^-)$	$1D(3/2^+)$	$1D(5/2^+)$
Σ_c	125.4°	-156.8°	124.8°	-151.4°	172.2°	-175.6°
Ξ_c'	125.0°	-153.6°	124.3°	-145.1°	168.9°	-173.8°

EHQ formula has been extended to study the decay properties of $1D$ Λ_c and Ξ_c states [11].

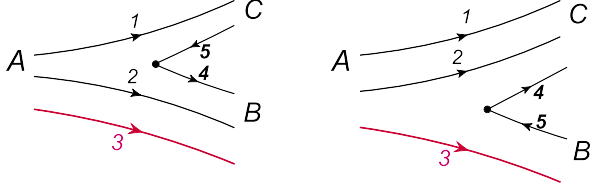
For the charmed baryons, the EHQ formula can be written as

$$\Gamma_{jc,\ell}^{A \rightarrow BC} = \xi \left(C_{jc,j_A,j_B}^{s_Q,j_B,j_A} \right)^2 \left| \mathcal{M}_{jc,\ell}^{j_A,j_B}(q) \right|^2 q^{2\ell+1} e^{-q^2/\tilde{\beta}^2}, \quad (16)$$

where ξ is the flavor factor given in Table XIII in Appendix B. $q = |\vec{q}|$ denotes the three-momentum of a final state in the rest frame of an initial state. A and B represent the initial and final heavy-light hadrons, respectively. C denotes the light flavor hadron (see Fig.3). The concrete expression of $\tilde{\beta}$ is given in

TABLE V: The meson effective β values in GeV.

States	Λ_c^+	Ξ_c	Σ_c	Ξ'_c
1S	1/2 ⁺	0.291	0.335	0.362
	3/2 ⁺		0.296	0.315
2S	0.145	0.162	0.144	0.152
3S	0.102	0.113	0.098	0.103
1P	0.184	0.205	0.182	0.192
2P	0.117	0.130	0.112	0.118
1D	0.142	0.156	0.136	0.143

FIG. 3: The two topological diagrams for an excited charmed baryon A decaying into the final states B and C . The brown line 3 denotes a charm quark.

Eq. (A11) in Appendix A. In addition, $C_{j_C, j_A, J_A}^{s_Q, j_B, J_B}$ is a normalized coefficient satisfying the following relation

$$C_{j_C, j_A, J_A}^{s_Q, j_B, J_B} = (-1)^{J_A + j_B + j_C + s_Q} \sqrt{(2j_A + 1)(2J_B + 1)} \times \left\{ \begin{matrix} s_Q & j_B & J_B \\ j_C & j_A & j_A \end{matrix} \right\}, \quad (17)$$

where $\vec{j}_C \equiv \vec{s}_C + \vec{\ell}$. The symbols s_C and ℓ represent the spin of the light hadron C and the orbital angular momentum relative to B , respectively. The transition factors $\mathcal{M}_{j_C, \ell}^{j_A, j_B}(q)$ involved in the concrete dynamics can only be calculated by various phenomenological models. For the decays of heavy-light mesons, transition factors have been calculated by the relativistic chiral quark model [68] and the 3P_0 model [63, 65, 69]. In our work, we will employ the 3P_0 model [70–72] to obtain the transition factors. More details for an estimate of the transition factors are given in Appendix A.

IV. DISCUSSION

A. Experimentally well established 1S and 1P states

At present, all the ground states and 1P \mathcal{G} -type charmed states have been experimentally established [1]. These states have been observed, at least, by two different collaborations, and their properties including masses and decays have been well determined. With good precision, the strong decays of these states provide a crucial test of our method.

Among the 1S charmed baryons, the measurements of $\Sigma_c(2455)$ and $\Sigma_c(2520)$ have been largely improved [2, 3] (see Table IX). In our calculation, the mass and decay width of

$\Sigma_c(2520)^{++}$ measured by CDF will be taken as input data to fix the constant γ peculiar to the 3P_0 model. With the transition factor of the process $\Sigma_c(2520) \rightarrow \Lambda_c(2286) + \pi$ (see Eq. (A12) in the Appendix A), the value of γ is fixed as 1.296.⁴

As shown in Tables VII and VIII, the predicted widths of other 1S charmed baryons are well consistent with experiments. Our results of mass spectra and decay widths indicate that $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Xi_c(2790)^{0,+}$, and $\Xi_c(2815)^{0,+}$ can be accommodated with the 1P \mathcal{G} -type charmed baryons. $\Lambda_c(2595)^+$ and $\Xi_c(2790)^{0,+}$ can be grouped as the $1/2^-$ states while $\Lambda_c(2625)^+$ and $\Xi_c(2815)^{0,+}$ as the $3/2^-$ states. The predicted mass splittings between the 1P $1/2^-$ and $3/2^-$ states are 25 MeV and 27 MeV for the Λ_c and Ξ_c baryons, respectively, which are also consistent with the experiments. The assignments of $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Xi_c(2790)^{0,+}$, and $\Xi_c(2815)^{0,+}$ are also supported by other works [9–12] in which the diquark scenario was also employed. In addition, the mass spectra obtained by different types of the quark potential models in the three-body picture also support these assignments [7, 8, 57–59]. However, the investigations by QCD sum rules indicate that these 1P candidates may have more complicated structures [14–16]. Especially, the work by Chen *et al.* suggested that $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ form the heavy doublet $\tilde{\Lambda}_{c1}(1/2^-, 3/2^-)$ (the same assignments as the case of $\Xi_c(2790)^{0,+}$ and $\Xi_c(2815)^{0,+}$) [16], which are different from our conclusion. Since the quantum numbers of J^P have not yet been determined for these 1P charmed states, more experiments are required in future.

B. 1P $\Sigma_c^{0,+,++}$ states

As shown in Tables II and III, the masses of 1P Σ_c states are predicted in the range of 2700–2800 MeV. Then, $\Sigma_c(2765)^+$ and $\Sigma_c(2800)^{0,+,++}$ can be grouped into the candidates of 1P Σ_c family. The predicted mass of $|1P, 1/2^- \rangle_h$ state is about 2765 MeV which is in good agreement with the measured mass of $\Sigma_c(2765)^+$. In addition, the theoretical result for the decay width of the $\Sigma_{c1}(1/2^-)$ state in Table X is about 63.52 MeV which is also in agreement with the measurements [30–32]. Furthermore, the signal of $\Sigma_c(2765)^+$ has been observed in the $\Sigma_c(2455)\pi$ intermediate states while there is no clear evidence for the decay of $\Sigma_c(2765)^+$ through $\Sigma_c(2520)\pi$ [31, 32]. This is also consistent with our results of the $|1P, 1/2^- \rangle_h$. Based on the combined analysis of the mass spectrum and strong decays, we, therefore, conclude that $\Sigma_c(2765)^+$ could be regarded as a good $\Sigma_{c1}(1/2^-)$ candidate. Considering uncertainties of the quark potential models, the masses obtained by Refs [9, 10, 57] are not contradictory to our assignment to $\Sigma_c(2765)^+$ here.

According to the predicted masses in Table III,

⁴ For the different conventions to extract the color and flavor factors, the value of γ here is different from these in Refs. [24, 74, 75]. Of course, the deviation does not effect the predictions since γ is regarded as an adjustable parameter in the 3P_0 model.

TABLE VI: The effective β values in GeV for the light diquark and various hadrons (the second row). The values of σ and C are given in the square brackets for various hadron structures (the third row).

$[qq]$	$\{qq\}$	$[qs]$	$\{qs\}$	π	ρ	K	K^*	D	D^*	p	Λ
0.201	0.143	0.207	0.159	0.298	0.179	0.291	0.201	0.250	0.230	0.189	0.226
[1.17, 0.39]		[1.57, 0.38]		[0.73, 0.63]		[0.83, 0.48]		[1.20, 0.63]		[- , 0.38]	
										[- , 0.26]	

TABLE VII: Open-flavor strong decay widths of $1S$ Σ_c and Ξ'_c in MeV.

$1S$ Σ_c and Ξ'_c					
$1/2^+$			$3/2^+$		
$\Sigma_c(2455)^{++}$	$\Xi'_c(2580)^+$		$\Sigma_c(2520)^{++}$	$\Xi_c(2645)^+$	
$\Lambda_c^+\pi^+$	1.53		$\Lambda_c^+\pi^+$ Input	$\Xi_c^0\pi^+$	1.54
				$\Xi_c^+\pi^0$	1.01
1.53	—		Input		2.55
$1.89^{+0.09}_{-0.18}$ [1]	—		14.9 ± 1.5 [1]	2.6 ± 0.6 [73]	

TABLE VIII: Open-flavor strong decay widths of $1P$ Λ_c and Ξ_c in MeV.

$1P$ Λ_c and Ξ_c					
$1/2^-$			$3/2^-$		
$\Lambda_c(2595)^+$	$\Xi_c(2790)^+$		$\Lambda_c(2625)^+$	$\Xi_c(2815)^+$	
$\Sigma_c\pi$	2.78	$\Xi'_c\pi$ 6.01	$\Sigma_c\pi$ 0.04	$\Xi'_c\pi$	0.15
				$\Xi_c^*\pi$	4.09
2.78	6.01		0.04		4.24
2.6 ± 0.6 [1]	8.9 ± 1.4 [4]		< 0.97 [1]	2.43 ± 0.37 [4]	

TABLE IX: The masses and widths (in units of MeV) of $\Sigma_c(2455)^{++}$ and $\Sigma_c(2520)^{++}$ measured by CDF [2] and Belle [3].

$\Sigma_c(2455)^{++}$	$2453.90 \pm 0.13 \pm 0.14$	2.34 ± 0.47	CDF
	$2453.97 \pm 0.01 \pm 0.02 \pm 0.14$	$1.84 \pm 0.04^{+0.07}_{-0.20}$	Belle
$\Sigma_c(2520)^{++}$	$2517.19 \pm 0.46 \pm 0.14$	15.03 ± 2.52	CDF
	$2518.45 \pm 0.10 \pm 0.02 \pm 0.14$	$14.77 \pm 0.25^{+0.18}_{-0.30}$	Belle

TABLE X: The partial and total decay widths of $1P$ Σ_c states (in units of MeV).

Decay modes	$1/2^- (1P)$		$3/2^- (1P)$		$5/2^- (1P)$
	$\Sigma_{c0}(2702)$	$\Sigma_{c1}(2765)$	$\Sigma_{c1}(2798)$	$\Sigma_{c2}(2785)$	$\Sigma_{c2}(2790)$
$\Lambda_c\pi$	3.64	×	×	24.06	24.63
$\Sigma_c(2455)\pi$	×	58.94	3.48	5.22	2.50
$\Sigma_c(2520)\pi$	×	1.70	63.72	2.47	4.34
$\Lambda_c(2595)\pi$		2.88	2.31	1.93	0.03
$\Lambda_c(2625)\pi$			3.12	0.07	0.63
Theory	3.64	63.52	72.63	33.75	32.13
Expt. [1]		≈ 50		72^{+22}_{-15}	

$\Sigma_c(2800)^{0,+,++}$ could be assigned to either $|1P, 3/2^- \rangle_l$, or $|1P, 3/2^- \rangle_h$, or $|1P, 5/2^- \rangle$ states. When we consider the decay properties of these three states (see Table X), the possibility of assignment to the $|1P, 3/2^- \rangle_l$ state can be excluded since the Belle Collaboration observed this state in the $\Lambda_c^+\pi$ mode.⁵ At present, the Belle Collaboration tentatively identified $\Sigma_c(2800)^{0,+,++}$ as members of the $\Sigma_{c2}(3/2^-)$ isospin triplet, which agrees with our results of both mass spectrum and strong decays. When the measured mass of $\Sigma_c(2800)^0$ is used for the $\Sigma_{c2}(3/2^-)$ state, the predicted width is about 40.1 MeV which is comparable with the experiment [34]. However, we notice that the quantum number J^P of $\Sigma_c(2800)^{0,+,++}$ has not been measured yet. Then the possibility of this state as the $\Sigma_{c2}(5/2^-)$ candidate can not be excluded by our results since the decay mode of $\Lambda_c^+\pi$ is dominant for this state. In addition, the predicted mass and total width of $\Sigma_{c2}(5/2^-)$ state are also possible for the $\Sigma_c(2800)^{0,+,++}$ baryon. Therefore, we would like to point out that the signal of $\Sigma_c(2800)^{0,+,++}$ found by Belle might be their overlapping structure. The future experiments may measure the following branching ratios to disentangle this state:

For the $\Sigma_{c2}(3/2^-)$ state,

$$\frac{\mathcal{B}(\Sigma_{c2}(3/2^-) \rightarrow \Sigma_c(2455)\pi)}{\mathcal{B}(\Sigma_{c2}(3/2^-) \rightarrow \Sigma_c(2520)\pi)} = 1.90; \quad (18)$$

$$\frac{\mathcal{B}(\Sigma_{c2}(3/2^-) \rightarrow \Lambda_c(2286)\pi)}{\mathcal{B}(\Sigma_{c2}(3/2^-) \rightarrow \Sigma_c(2455)\pi)} = 4.07 \quad (19)$$

For the $\Sigma_{c2}(5/2^-)$ state,

$$\frac{\mathcal{B}(\Sigma_{c2}(5/2^-) \rightarrow \Sigma_c(2455)\pi)}{\mathcal{B}(\Sigma_{c2}(5/2^-) \rightarrow \Sigma_c(2520)\pi)} = 0.58. \quad (20)$$

$$\frac{\mathcal{B}(\Sigma_{c2}(5/2^-) \rightarrow \Lambda_c(2286)\pi)}{\mathcal{B}(\Sigma_{c2}(5/2^-) \rightarrow \Sigma_c(2455)\pi)} = 9.85 \quad (21)$$

As mentioned earlier, the signal $\Sigma_c(2850)^0$ discovered by the BaBar collaboration may be a $J = 1/2$ state. If $\Sigma_c(2850)^0$

⁵ Even the possible mixing between $\Sigma_{c1}(3/2^-)$ and $\Sigma_{c2}(3/2^-)$ is considered, the partial width of $\Lambda_c^+\pi$ is only 3.87 MeV for the $|1P, 3/2^- \rangle_l$ state where the mixing angle obtained in Section II has been used.

is the $1/2^+(2S)$ state, the corresponding ratios (see Subsection IV E) are different from Eqs. (18–21). So the measurements of these ratios of branching fractions can help us understand the nature of $\Sigma_c(2800)^{0,+,++}$ and $\Sigma_c(2850)^0$.

Although at present, the $\Sigma_{c0}(1/2^-)$ and $\Sigma_{c1}(3/2^-)$ states are still missing in experiments, our results indicate that the $\Sigma_{c0}(1/2^-)$ state may be a narrow resonance and its predominant decay channel $\Lambda_c^+\pi$ is only about 3.64 MeV (see Table X). Since the decay mode, $\Sigma_c(2520)\pi$, is the largest for the $\Sigma_{c1}(3/2^-)$ state, we suggest to search this channel for this state in the future experiments. In the heavy quark limit, the above branching ratio for $\Sigma_{c1}(3/2^-)$ state is much smaller than $\Sigma_{c2}(3/2^-)$, *i.e.*,

$$\frac{\mathcal{B}(\Sigma_{c1}(3/2^-) \rightarrow \Sigma_c(2455)\pi)}{\mathcal{B}(\Sigma_{c1}(3/2^-) \rightarrow \Sigma_c(2520)\pi)} = 0.04. \quad (22)$$

C. $1P \Xi_c'^{0,+}$ states

TABLE XI: The partial and total decay widths of $1P \Xi_c'$ states (in units of MeV).

Decay modes	$1/2^- (1P)$		$3/2^- (1P)$		$5/2^- (1P)$
	$\Xi_{c0}'(2839)$	$\Xi_{c1}'(2900)$	$\Xi_{c1}'(2932)$	$\Xi_{c2}'(2921)$	$\Xi_{c2}'(2927)$
$\Lambda_c K$	46.59	×	×	11.59	12.43
$\Xi_c \pi$	4.39	×	×	7.42	7.75
$\Xi_c'(2580)\pi$	×	9.44	0.76	1.20	0.57
$\Xi_c'(2645)\pi$	×	0.52	3.23	0.75	1.31
$\Xi_c(2790)\pi$			0.01		
Theory	50.98	9.96	4.00	20.96	22.06
Expt.				$36 \pm 7 \pm 11$ [36]	

As shown in Table III, the predicted masses of $1P \Xi_c'$ is in the range from 2840 to 2930 MeV. Then the resonance structure observed by BaBar [36] in the decay channel $B^- \rightarrow \Xi_c'(2930)^0 \bar{\Lambda}_c^- \rightarrow \Lambda_c^+ K^- \bar{\Lambda}_c^-$ with an invariant mass of 2.93 GeV could be a good candidate of $1P \Xi_c'$ members. The results of decays in Table XI favor $\Xi_c'(2930)^0$ as the $\Xi_{c2}'(3/2^-)$ or $\Xi_{c2}'(5/2^-)$ state. Then $\Xi_c'(2930)^0$ might be regarded as the strange partner of $\Sigma_c(2800)^{0,+,++}$ by our results. Interestingly, the mass difference between $\Xi_c'(2930)^0$ and $\Sigma_c(2800)^{0,+,++}$ is about 130 MeV which is comparable with the mass differences among sextet states of ground charmed baryons [21]. With a chiral quark model, Liu *et al.* also analyzed the $\Xi_c'(2930)^0$ by the two-body strong decays [23]. Their results supported $\Xi_c'(2930)^0$ as the $|\Xi_c'^2 P_\lambda, 1/2^- \rangle$ or the $|\Xi_c'^4 P_\lambda, 1/2^- \rangle$ state. Since the heavy quark symmetry has not been considered in Ref [23], the notations of charm-strange baryons in Ref [23] are different from our $\Xi_{c0}'(1/2^-)$ and $\Xi_{c1}'(1/2^-)$. Although the results in Table XI indicate that the $\Lambda_c^+ K$ decay mode dominates the decay of $\Xi_{c0}'(1/2^-)$ state, the mass of this state is about 2840 MeV which is much smaller than $\Xi_c^0(2930)$. In addition, the $\Lambda_c^+ K$ decay mode is forbidden for the $\Xi_{c1}'(1/2^-)$ state. Thus, $\Xi_c^0(2930)$ is unlikely to be a $1P$ state with $J^P = 1/2^-$ due to our results.

Another charm-strange baryon, $\Xi_c(2980)^{0,+}$, is slightly higher than the predicted mass range of $1P \Xi_c'$ states. This state has been observed in $\Sigma_c(2455)K$, $\Xi_c'(2645)\pi$, and non-resonant $\Lambda_c^+ \bar{K}\pi$ decay channels. However, it was not seen in the decay modes of $\Lambda_c^+ \bar{K}$ and $\Xi_c \pi$ [37–39]. Comparing the mass and decay properties of $\Xi_c(2980)^{0,+}$ with our results, the possibility as a $1P \Xi_c'$ state might be excluded. As shown in the next Subsection, $\Xi_c(2980)^{0,+}$ could be a good $2S \Xi_c$ candidate. Based on our results on strong decays, we find that the $\Xi_{c1}'(1/2^-)$ and $\Xi_{c1}'(3/2^-)$ are quite narrow (see Table XI).

D. $2S \Lambda_c^+$ and $\Xi_c'^{0,+}$ states

According to the mass spectrum, $\Lambda_c/\Sigma_c(2765)^+$ can also be regarded as the first radial ($2S$) excitation of the $\Lambda_c(2286)^+$ with $J^P = 1/2^+$. Interestingly, the results of strong decays in Table XII do not contradict with this assignment. Our calculation indicates that the decay channel $\Sigma_c(2455)\pi$ is a dominant decay channel for the $\Lambda_c^+(2S)$ state. This is in line with the observations by Belle [31, 32]. At present, both $1/2^+(2S) \Lambda_c^+$ and $1/2^-(1P) \Sigma_c^+$ are possible for the assignment of $\Lambda_c/\Sigma_c(2765)^+$. However, there is a very important discriminator for experiments to distinguish these two assignments in future. Specifically, we suggest to search $\Lambda_c/\Sigma_c(2765)^+$ in the channel of $\Sigma_c(2520)\pi$. As shown in Table XII, the channel $\Sigma_c(2520)\pi$ is large enough to find the $\Lambda_c^+(2S)$ state. Differently, this mode seems too small to be detected for the $\Sigma_{c1}(1/2^-)$ (see Table X). Explaining the criteria concretely, we give the following branching ratios for these two states,

For the $\Lambda_c(2S)$ state,

$$\frac{\mathcal{B}(\Lambda_c(2765) \rightarrow \Sigma_c(2520)\pi)}{\mathcal{B}(\Lambda_c(2765) \rightarrow \Sigma_c(2455)\pi)} = 0.74; \quad (23)$$

For the $\Sigma_{c1}(1/2^-)$ state,

$$\frac{\mathcal{B}(\Sigma_{c1}(2765) \rightarrow \Sigma_c(2520)\pi)}{\mathcal{B}(\Sigma_{c1}(2765) \rightarrow \Sigma_c(2455)\pi)} = 0.03. \quad (24)$$

The branching ratio of $\mathcal{B}(\Sigma_c(2520)\pi)/\mathcal{B}(\Sigma_c(2455)\pi)$ for the $\Sigma_{c1}(1/2^-)$ state is roughly an order of magnitude smaller than $\Lambda_c(2S)$. If $\Lambda_c(2765)$ is the $2S$ excitation, $\Xi_c(2980)$ could be a good candidate as its charm-strange analog [21] as seen in Table XII. The mass difference between $\Lambda_c(2765)$ and $\Xi_c(2980)$ is about 200 MeV which nearly equals the mass difference between $\Lambda_c(2287)$ and $\Xi_c(2470)$. The predicted width of $\Xi_c(2980)$ is 27.44 MeV which is in good agreement with the experimental data [4, 39]. As the $2S$ excitation of $\Xi_c(2470)$, the branching ratio below

$$\frac{\mathcal{B}(\Xi_c(2980) \rightarrow \Xi_c'(2580)\pi)}{\mathcal{B}(\Xi_c(2980) \rightarrow \Xi_c(2645)\pi)} = 0.89, \quad (25)$$

is predicted for $\Xi_c(2980)$, which can be tested by the future experiments. Recently, the following ratio of branching fraction

$$\frac{\mathcal{B}(\Xi_c(2980)^+ \rightarrow \Xi_c'(2580)^0 \pi^+)}{\mathcal{B}(\Xi_c(2815)^+ \rightarrow \Xi_c(2645)^0 \pi^+ \rightarrow \Xi_c^+ \pi^- \pi^+)} \approx 75\%, \quad (26)$$

has been estimated by Belle Collaboration [4]. Combining with the predicted partial widths of $\Xi_c(2815)$ and $\Xi_c(2645)$ in Table VII and VIII, the branching fraction $\mathcal{B}(\Xi_c(2980)^+ \rightarrow \Xi'_c(2580)^0 \pi^+)$ is evaluated about 40% which is consistent with our result of 41.8%.

E. $2S$ $\Sigma_c^{0,+,++}$ and $\Xi_c^{0,+}$ states

In Table III, masses of the $2S$ $\Sigma_c(1/2^+, 3/2^+)$ states are predicted as 2850 MeV and 2876 MeV, respectively. The neutral $\Sigma_c(2850)^0$ which was found by the BaBar Collaboration in the decay channel $B^- \rightarrow \Sigma_c(2850)^0 \bar{p} \rightarrow \Lambda_c^+ \pi^- \bar{p}$ [35] can be regarded as the $2S$ Σ_c state with $J^P = 1/2^+$. The mass and width of the neutral $\Sigma_c(2800)^0$ and $\Sigma_c(2850)^0$ are collected below.

$$\begin{aligned} \Sigma_c(2800)^0: \quad m &= 2806_{-7}^{+5} \text{ MeV}, \quad \Gamma = 72_{-15}^{+22} \text{ MeV}; \\ \Sigma_c(2850)^0: \quad m &= 2846 \pm 8 \pm 10 \text{ MeV}, \quad \Gamma = 86_{-22}^{+33} \text{ MeV}. \end{aligned}$$

For lack of experimental information, at present, PDG treated $\Sigma_c(2850)^0$ and $\Sigma_c(2800)^{0,+,++}$ as the same state [1]. As pointed out by the BaBar collaboration [35], however, there are indications that these two signals detected by Belle [34] and BaBar [35] are two different Σ_c^* states. The main reasons are listed as follows:

1. Although the widths of $\Sigma_c(2800)^{0,+,++}$ and $\Sigma_c(2850)^0$ are consistent with each other, their masses are 3σ apart.
2. The Belle Collaboration tentatively identified the $\Sigma_c(2800)^{0,+,++}$ as the $J = 3/2$ isospin triple, while the BaBar Collaboration found the weak evidence of $\Sigma_c(2850)^0$ as a $J = 1/2$ state.

Our results also indicate that $\Sigma_c(2800)^{0,+,++}$ and $\Sigma_c(2850)^0$ are the different Σ_c excited states. One notices that the predicted mass of $1/2^+(2S)$ Σ_c state in this work and in Ref. [9] are around 2850 MeV. Even the results in Refs. [10, 57] are only about 50 MeV larger than the measurements. Due to the intrinsic uncertainties of the quark potential model, it is appropriate to assign $\Sigma_c(2850)^0$ as a $2S$ $1/2^+$ state. More importantly, the predicted decay width of $\Sigma_c(1/2^+, 2S)$ state is 118.36 MeV which is comparable with the measurement by BaBar [35]. The partial width of $\Lambda_c \pi$ is 35.11 MeV, which can explain why $\Sigma_c(2850)^0$ was first found in this channel. We find that the decay modes of $\Sigma_c(2455)\pi$ and $\Sigma_c(2520)\pi$ are also large. Finally, we give the following branching ratio,

$$\frac{\mathcal{B}(\Sigma_c(2850) \rightarrow \Sigma_c(2455) \pi)}{\mathcal{B}(\Sigma_c(2850) \rightarrow \Sigma_c(2520) \pi)} = 3.26 \quad (27)$$

and

$$\frac{\mathcal{B}(\Sigma_c(2850) \rightarrow \Sigma_c(2286) \pi)}{\mathcal{B}(\Sigma_c(2850) \rightarrow \Sigma_c(2455) \pi)} = 0.61, \quad (28)$$

which can be tested by the future experiments. If $\Sigma_c(2850)^0$ is the $1/2^+(2S)$ state, the mass of its doublet partner in the heavy quark effective theory is predicted as 2876 MeV (denoted as $\Sigma_c(2880)$). According to the predicted decay widths

in Table XII, this state might be also broad. $\Lambda_c^+ \pi$, $\Sigma_c(2455)\pi$, and $\Sigma_c(2520)\pi$ are also dominant for $\Sigma_c(2880)$. The ratio of $\Gamma(\Sigma_c(2455)\pi)/\Gamma(\Sigma_c(2520)\pi)$ for $\Sigma_c(2880)$ is different from $\Sigma_c(2850)$ and its numerical value is given by,

$$\frac{\mathcal{B}(\Sigma_c(2880) \rightarrow \Sigma_c(2455) \pi)}{\mathcal{B}(\Sigma_c(2880) \rightarrow \Sigma_c(2520) \pi)} = 0.29. \quad (29)$$

Even though the strange partners of $\Sigma_c(2850)$ and $\Sigma_c(2880)$ have not been found by any experiments, their decay properties are calculated and presented in Table XII. Our results indicate that $\Lambda_c^+ K$, $\Xi'_c(2580)\pi$, and $\Sigma_c(2455)K$ are the dominant decay modes of the $\Xi'_c(3000)$ state with $J^P = 1/2^+$, while $\Lambda_c^+ K$ and $\Xi'_c(2645)\pi$ of the $\Xi'_c(3030)$.⁶ Besides the masses and decay widths, the following branching ratios may also be valuable for the future experiments:

$$\frac{\mathcal{B}(\Xi'_c(3000) \rightarrow \Xi'_c(2580) \pi)}{\mathcal{B}(\Xi'_c(3000) \rightarrow \Xi'_c(2645) \pi)} = 3.04, \quad (30)$$

and

$$\frac{\mathcal{B}(\Xi'_c(3030) \rightarrow \Xi'_c(2580) \pi)}{\mathcal{B}(\Xi'_c(3030) \rightarrow \Xi'_c(2645) \pi)} = 0.27. \quad (31)$$

V. SUMMARY AND OUTLOOK

Up to now, several candidates of the $1P$ and $2S$ charm and charm-strange baryons have been found by experiments, and some of them are still open to debate. To better understand these low excited charmed baryons, in this paper, we carry a systematical research of the mass spectra and strong decays for the $1P$ and $2S$ charmed baryon states in the framework of nonrelativistic constituent quark model. The masses have been calculated in the potential model where the charmed baryons are treated simply as a quasi two body system in a diquark picture. The strong decays are computed by the EHQ decay formula [67] where the transition factors are determined by the 3P_0 model. When calculating the decays, the inner structure of a diquark has also been considered. Except for the unique parameter γ of the QPC model, the parameters in the potential model and in the EHQ decay formula have the same values.

The well-established ground and $1P$ \mathcal{G} -type charmed baryons provide a good test to our method. The experimental properties including both masses and widths for these states can be well explained by our results. This success has made us more confident of the predictions for other $1P$ and $2S$ states. Our main conclusions are given as follows:

The broad state $\Lambda_c(2765)^+$ (or $\Sigma_c(2765)^+$) which is still ambiguous could be assigned to the $1/2^+(2S)$ Λ_c^+ , or the $1/2^-(1P)$ Σ_{c1}^+ state. The branching ratio

⁶ If $\Xi_c(2980)$ is the first radial excited state of $\Xi_c(2470)$. Then our predicted masses for $2S$ charm-strange baryons may be about 20~30 MeV lower than experiments. For this purpose, we increase about 25 MeV for the $2S$ Ξ'_c states

TABLE XII: The partial and total decay widths of $2S$ Λ_c^+ and $\Xi_c^{+,0}$ states (in units of MeV).

$1/2^+ (2S)$				$1/2^+ (2S)'$				$3/2^+ (2S)'$			
$\Lambda_c(2765)^+$		$\Xi_c(2980)$		$\Sigma_c(2850)^0$		$\Xi'_c(3000)$		$\Sigma_c(2880)^0$		$\Xi'_c(3030)$	
$\Sigma_c(2455)\pi$	26.23	$\Sigma_c(2455)K$	3.14	$\Lambda_c^+\pi$	35.11	Λ_c^+K	17.42	$\Lambda_c^+\pi$	34.96	Λ_c^+K	18.37
$\Sigma_c(2520)\pi$	19.28	$\Xi'_c(2580)\pi$	11.47	$\Sigma_c(2455)\pi$	57.16	$\Xi'_c(2580)\pi$	12.56	$\Sigma_c(2455)\pi$	15.98	$\Xi'_c(2580)\pi$	3.50
		$\Xi'_c(2645)\pi$	12.83	$\Sigma_c(2520)\pi$	17.54	$\Xi'_c(2645)\pi$	4.13	$\Sigma_c(2520)\pi$	54.52	$\Xi'_c(2645)\pi$	12.92
				$\Lambda_c(2595)\pi$	6.92	$\Xi_c(2790)\pi$	5.89	$\Lambda_c(2595)\pi$	1.07	$\Xi_c(2790)\pi$	0.40
				$\Lambda_c(2625)\pi$	1.57	$\Xi_c(2815)\pi$	0.13	$\Lambda_c(2625)\pi$	7.62	$\Xi_c(2815)\pi$	6.22
				D^0n	0.03	$\Sigma_c(2455)K$	15.34	D^0n	3.03	$\Sigma_c(2455)K$	6.49
										$D^0\Lambda$	0.01
45.51		27.44		118.33		55.47		117.18		47.91	
≈ 50 [1]		$28.1 \pm 2.4^{+1.0}_{-5.0}$ [4]		86^{+33}_{-22} [35]							

$\mathcal{B}(\Sigma_c(2455)\pi)/\mathcal{B}(\Sigma_c(2520)\pi)$ is found to be different for these two assignments, which may help us understand the nature of this state.

$\Sigma_c(2800)^{0,+,++}$ observed by the Belle Collaboration in e^+e^- annihilation processes [34] can be regarded as a negative parity state with $J^P = 3/2^-$, or $5/2^-$, or their overlapping structure. We suggest to measure the $\mathcal{B}(\Sigma_c(2455)\pi)/\mathcal{B}(\Sigma_c(2520)\pi)$ in future. Another neutral state, $\Sigma_c(2850)^0$, which was found in the B^- meson decay [35] could be a good candidate for the first radial excited state of $\Sigma_c(2455)$. If $\Sigma_c(2850)^0$ is a $1/2^+(2S)$ state, the ratio $\mathcal{B}(\Sigma_c(2455)\pi)/\mathcal{B}(\Sigma_c(2520)\pi)$ is predicted to be 3.26. With the above assignments, the ratios of $\mathcal{B}(\Lambda_c(2287)\pi)/\mathcal{B}(\Sigma_c(2455)\pi)$ shall be very different for $\Sigma_c(2800)^{0,+,++}$ and $\Sigma_c(2850)^0$, i.e., 4.07 for $\Sigma_c(2800)^{0,+,++}$ and 0.61 for $\Sigma_c(2850)^0$. The puzzles of $\Sigma_c(2800)^{0,+,++}$ and $\Sigma_c(2850)^0$ may be disentangled if these branching ratios are measured in future.

The values of the mass and decay properties support that $\Xi_c(2980)^{0,+}$ is the $2S$ excitation (the first radial excited state of $\Xi_c(2470)$). The existence of $\Xi_c(2930)^0$ is still in dispute. If it exists, the assignments of $\Xi'_c(3/2^-)$ and $\Xi'_c(5/2^-)$ are possible. In other words, it could be regarded as a strange partner of $\Sigma_c(2800)^{0,+,++}$. Some useful ratios of partial decay widths are also presented for $\Xi_c(2980)^{0,+}$ and $\Xi_c(2930)^0$.

Although both the masses and strong decays have been explained in the heavy quark-light diquark picture for the observed $2S$ and $1P$ candidates, it is not the end of the story to study the excited charmed baryon states. Investigation of the ρ mode excited states with higher energies are also important to identify the effective degrees of freedom of charmed baryons. However, this topic needs much laborious work and is beyond the scope of the present work. In addition, the quark model employed here neglects the effect of virtual hadronic loops. In future, a more reasonable scheme for studying the properties of heavy baryons will be achieved by the unquenched quark model. Another topic which is left as a future task is to calculate the sum rules among the branching fractions of charmed baryons by applying the technique found in Ref. [79].

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Appendix A: Transition factor $\mathcal{M}_{jc,\ell}^{iA,jB}(q)$ in the QPC model

Usually, there are two possible decay processes for an excited charmed baryon state (see Fig. 3). The final states of the left figure contain a charmed baryon and a light meson. The right one contains a charmed meson and a light baryon. If a baryon decays via the so-called 3P_0 mechanism, a quark-antiquark pair is created from the vacuum and then regroups two outgoing hadrons by a quark rearrangement process. In the non-relativistic limit, the transition operator $\hat{\mathcal{T}}$ of the 3P_0 model is given by

$$\begin{aligned} \hat{\mathcal{T}} = & -3\gamma \sum_{m,i,j} \langle 1, m; 1, -m | 0, 0 \rangle \iint d^3\vec{k}_4 d^3\vec{k}_5 \delta^3(\vec{k}_4 + \vec{k}_5) \\ & \times \mathcal{Y}_1^m \left(\frac{\vec{k}_4 - \vec{k}_5}{2} \right) \omega_{i,j}^{(4,5)} \varphi_0^{(4,5)} \chi_{1,-m}^{(4,5)} d_{4i}^\dagger(\vec{k}_4) d_{5j}^\dagger(\vec{k}_5), \end{aligned} \quad (\text{A1})$$

where the $\omega_{i,j}^{(4,5)}$ and $\varphi_0^{(4,5)}$ are the color and flavor wave functions of the $q_4\bar{q}_5$ pair created from the vacuum. Thus, $\omega_{i,j}^{(4,5)} = (R\bar{R} + G\bar{G} + B\bar{B})/\sqrt{3}$, $\varphi_0^{(4,5)} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ are color and flavor singlets. The pair is also assumed to carry the quantum number of 0^{++} , suggesting that they are in a 3P_0 state. The $\chi_{1,-m}^{(4,5)}$ represents the pair production in a spin triplet state. The solid harmonic polynomial $\mathcal{Y}_1^m(\vec{k}) \equiv |\vec{k}| \mathcal{Y}_1^m(\theta_k, \phi_k)$ reflects the momentum-space distribution of the $q_4\bar{q}_5$. γ is a dimensionless constant which denotes the strength of the quark-antiquark pair created from the vacuum. The value of

γ is usually fixed by fitting the well measured partial decay widths.

When the mock state [76] is adopted to describe the spatial wave function of a meson, the helicity amplitude $\mathcal{M}^{M_{JA}, M_{JB}, M_{JC}}(q)$ can be easily constructed in the L - S basis [72]. The mock state for an A meson is

$$|A(n_A^{2S_A+1} L_A^{J_A M_{JA}}(\vec{P}_A))\rangle \equiv \omega_A^{123} \phi_A^{123} \prod_A \int d^3 \vec{k}_1 d^3 \vec{k}_2 d^3 \vec{k}_3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{P}_A) \times \Psi_{n_A}^{L_A M_{LA}}(\vec{k}_1, \vec{k}_2, \vec{k}_3) |q_1(\vec{k}_1) q_2(\vec{k}_2) q_3(\vec{k}_3)\rangle. \quad (A2)$$

As for the left decay process in Fig. 3, the wave function of a B baryon can be constructed in the same way. The wave function of a C meson is

$$|C(n_C^{2S_C+1} L_C^{J_C M_{JC}}(\vec{P}_C))\rangle \equiv \omega_A^{15} \phi_A^{15} \prod_C \int d^3 \vec{k}_1 d^3 \vec{k}_5 \times \delta^3(\vec{k}_1 + \vec{k}_5 - \vec{P}_C) \psi_{n_C}^{L_C M_{LC}}(\vec{k}_1, \vec{k}_5) |q_1(\vec{k}_1) \bar{q}_5(\vec{k}_5)\rangle. \quad (A3)$$

Here, the symbols of \prod_i ($i = A, B$, and C) represent the Clebsch-Gordan coefficients for the initial and final hadrons, which arise from the couplings among the orbital, spin, and total angular momentum and their projection of l_z and s_z to j_z . More specifically, \prod_i ($i = A, B$, and C) are given by

$$\begin{aligned} &\langle s_1 m_1, s_2 m_2 | s_{12} m_{12} \rangle \langle s_{12} m_{12}, s_3 m_3 | S_A s_A \rangle \langle L_A l_A, S_A s_A | J_A j_A \rangle; \\ &\langle s_2 m_2, s_5 m_5 | s_{25} m_{25} \rangle \langle s_{25} m_{25}, s_3 m_3 | S_B s_B \rangle \langle L_B l_B, S_B s_B | J_B j_B \rangle; \\ &\langle s_1 m_1, s_4 m_4 | s_{14} m_{14} \rangle \langle s_{14} m_{14}, s_5 m_5 | S_C s_C \rangle \langle L_C l_C, S_C s_C | J_C j_C \rangle, \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{LS}(q) = & -3\gamma \sum_{l_i, m_j} \langle LO; J j | J_A j_A \rangle \langle J_B j_B; J_C j_C | J j \rangle \langle s_1 m_1; s_2 m_2 | s_{dA} m_{12} \rangle \langle s_{dA} m_{12}; s_3 m_3 | S_A s_A \rangle \langle S_A s_A; L_A l_A | J_A j_A \rangle \langle s_2 m_2; s_5 m_5 | s_{dB} m_{25} \rangle \\ & \langle s_{dB} m_{25}; s_3 m_3 | S_B s_B \rangle \langle S_B s_B; L_B l_B | J_B j_B \rangle \langle s_1 m_1; s_4 m_4 | s_{14} m_{14} \rangle \langle s_{14} m_{14}; s_5 m_5 | S_C s_C \rangle \langle L_C l_C | J_C j_C \rangle \langle s_4 m_4; s_5 m_5 | 1 - m \rangle \langle 1, m; 1, -m | 0, 0 \rangle \\ & \langle \varphi_B^{235} \varphi_C^{14} | \varphi_0^{45} \varphi_A^{123} \rangle \langle \omega_B^{235} \omega_C^{14} | \omega_0^{45} \omega_A^{123} \rangle \int \dots \int d^3 \vec{k}_1 \dots d^3 \vec{k}_5 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \delta^3(\vec{q} - \vec{k}_1 - \vec{k}_4) \delta^3(\vec{q} + \vec{k}_2 + \vec{k}_3 + \vec{k}_5) \\ & \delta^3(\vec{k}_4 + \vec{k}_5) \Psi_A(\vec{k}_1, \vec{k}_2, \vec{k}_3) \Psi_B^*(\vec{k}_1, \vec{k}_2, \vec{k}_4) \psi_C^*(\vec{k}_3, \vec{k}_5) \mathcal{Y}_1^m\left(\frac{\vec{k}_3 - \vec{k}_4}{2}\right), \end{aligned} \quad (A7)$$

where, $i = A, B, C$ and $j = 1, 2, \dots, 5$. The color matrix element $\langle \omega_B^{235} \omega_C^{14} | \omega_0^{45} \omega_A^{123} \rangle$ is a constant which can be absorbed into the parameter γ . The flavor matrix element $\xi = \langle \varphi_B^{235} \varphi_C^{14} | \varphi_0^{45} \varphi_A^{123} \rangle$ will be presented in the next Subsection. To obtain the analytical amplitudes, the SHO wave functions are employed to describe the spatial wave function of a hadron. In the momentum space, the SHO radial wave func-

tion, $\psi_{Lm}^n(\mathbf{q})$, is given by

The helicity amplitude $\mathcal{M}^{M_{JA}, M_{JB}, M_{JC}}(q)$ is defined by

$$\langle BC | \hat{T} | A \rangle = \delta^3(\vec{P}_A - \vec{P}_B - \vec{P}_C) \mathcal{M}^{M_{JA}, M_{JB}, M_{JC}}(q), \quad (A4)$$

where q represents the momentum of an outgoing meson in the rest frame of a meson A . For comparison with experiments, one obtains the partial decay width $\mathcal{M}_{LS}(q)$ via the Jacob-Wick formula [77]

$$\mathcal{M}_{LS}(q) = \frac{\sqrt{2L+1}}{2J_A+1} \sum_{M_{JB}, M_{JC}} \langle LO J M_{JA} | J_A M_{JA} \rangle \times \langle J_B, M_{JB} J_C, M_{JC} | J M_{JA} \rangle \mathcal{M}^{M_{JA}, M_{JB}, M_{JC}}(q). \quad (A5)$$

Then the decay width $\Gamma(A \rightarrow BC)$ is derived analytically in terms of the partial wave amplitudes in the A rest frame,

$$\Gamma(A \rightarrow BC) = 2\pi \frac{E_B E_C}{M_A} q \sum_{LS} |\mathcal{M}_{LS}(q)|^2. \quad (A6)$$

Finally, the full expression of $\mathcal{M}_{LS}(q)$ in the rest frame of the baryon A is

tion, $\psi_{Lm}^n(\mathbf{q})$, is given by

$$\begin{aligned} \psi_{Lm}^n(\mathbf{q}) &= \frac{(-1)^n}{\beta^{3/2}} \sqrt{\frac{2(2n-1)!}{\Gamma(n+L+\frac{1}{2})}} \left(\frac{q}{\beta}\right)^L e^{-\frac{q^2}{2\beta^2}} L_{n-1}^{L+1/2}\left(\frac{q^2}{\beta^2}\right) \mathcal{Y}_{Lm}(\mathbf{q}), \end{aligned} \quad (A8)$$

with $\mathbf{q} = (m_i \vec{k}_j - m_j \vec{k}_i)/(m_i + m_j)$ and $\mathcal{Y}_{Lm}(\mathbf{q}) = |\mathbf{q}|^L Y_{Lm}(\Omega_{\mathbf{q}})$. The values of the SHO wave function scale parameter β have been given in Table V. In the diquark picture, the wave func-

tion of a charmed baryon can be easily constructed. Taking the A baryon as an example, the wave functions corresponding to the $1S$, $2S$, and $1P$ states are given as follows, respectively,

$$\begin{aligned}\Psi_{00}^0 &= \frac{3^{3/4}}{\pi^{3/2}\beta_{dA}^{3/2}\beta_A^{3/2}} e^{-\frac{1}{2\beta_{dA}^2}\left(\frac{m_1\vec{k}_2-m_2\vec{k}_1}{m_1+m_2}\right)^2 - \frac{1}{2\beta_A^2}\left[\frac{(m_1+m_2)\vec{k}_3-m_Q(\vec{k}_1+\vec{k}_2)}{m_1+m_2+m_Q}\right]^2}; \\ \Psi_{00}^1 &= -\frac{3^{3/4}}{\sqrt{6}\pi^{3/2}\beta_{dA}^{3/2}\beta_A^{3/2}} e^{-\frac{1}{2\beta_{dA}^2}\left(\frac{m_1\vec{k}_2-m_2\vec{k}_1}{m_1+m_2}\right)^2 - \frac{1}{2\beta_A^2}\left[\frac{(m_1+m_2)\vec{k}_3-m_Q(\vec{k}_1+\vec{k}_2)}{m_1+m_2+m_Q}\right]^2} \\ &\quad \times \left\{ 3 - \frac{2}{\beta_A^2} \left[\frac{(m_1+m_2)\vec{k}_3-m_Q(\vec{k}_1+\vec{k}_2)}{m_1+m_2+m_Q} \right]^2 \right\}; \\ \Psi_{1m}^0 &= \frac{3^{3/4} \times 2\sqrt{2/3}}{\pi\beta_{dA}^{3/2}\beta_A^{5/2}} e^{-\frac{1}{2\beta_{dA}^2}\left(\frac{m_1\vec{k}_2-m_2\vec{k}_1}{m_1+m_2}\right)^2 - \frac{1}{2\beta_A^2}\left[\frac{(m_1+m_2)\vec{k}_3-m_Q(\vec{k}_1+\vec{k}_2)}{m_1+m_2+m_Q}\right]^2} \\ &\quad \times \mathcal{Y}_{1m}\left(\frac{(m_1+m_2)\vec{k}_3-m_Q(\vec{k}_1+\vec{k}_2)}{m_1+m_2+m_Q}\right).\end{aligned}$$

With the help of Eq. (A7), the transition amplitude can be simplified. In the following, we take the process $\Sigma_c(2520) \rightarrow \Lambda_c(2280)^+\pi$ as an example. The wave functions of initial and final states are

$$\begin{aligned}\Psi_A &= \frac{3^{3/4}}{\pi^{3/2}\beta_{dA}^{3/2}\beta_A^{3/2}} e^{-\frac{1}{2\beta_{dA}^2}\left(\frac{m_1\vec{k}_2-m_2\vec{k}_1}{m_1+m_2}\right)^2 - \frac{1}{2\beta_A^2}\left[\frac{(m_1+m_2)\vec{k}_3-m_3(\vec{k}_1+\vec{k}_2)}{m_1+m_2+m_3}\right]^2}; \\ \Psi_B &= \frac{3^{3/4}}{\pi^{3/2}\beta_{dB}^{3/2}\beta_B^{3/2}} e^{-\frac{1}{2\beta_{dB}^2}\left(\frac{m_4\vec{k}_2-m_5\vec{k}_1}{m_2+m_5}\right)^2 - \frac{1}{2\beta_B^2}\left[\frac{(m_2+m_5)\vec{k}_3-m_Q(\vec{k}_2+\vec{k}_5)}{m_2+m_3+m_5}\right]^2}; \\ \psi_C &= -\frac{1}{\pi^{3/4}\beta_C^{3/2}} e^{-\frac{1}{2\beta_C^2}\left(\frac{m_1\vec{k}_4-m_4\vec{k}_1}{m_1+m_2}\right)^2}.\end{aligned}$$

Based on Eq. (A7), we obtain the amplitude as

$$\mathcal{M}_{1\frac{1}{2}}(q) = -\frac{3g}{8\pi^{5/4}f^{5/2}\lambda^{3/2}\beta_A^{3/2}\beta_{dA}^{3/2}\beta_B^{3/2}\beta_{dB}^{3/2}\beta_C^{3/2}} p e^{-\frac{4fg-k^2}{4f}q^2}. \quad (\text{A9})$$

where

$$\begin{aligned}f &= \frac{1}{2\beta_{dA}^2} + \frac{1}{2\beta_{dB}^2} + \frac{1}{2\beta_M^2} - \frac{\mu^2}{4\lambda}; \\ g &= \frac{1}{\beta_{dA}^2} + \frac{\varepsilon_3}{\beta_{dB}^2} + \frac{\varepsilon_4}{\beta_M^2} - \frac{\mu\nu}{2\lambda}; \\ h &= \frac{1}{2\beta_{dA}^2} + \frac{\varepsilon_2^2}{2\beta_B^2} + \frac{\varepsilon_3^2}{2\beta_{dB}^2} + \frac{\varepsilon_4^2}{2\beta_M^2} - \frac{\nu^2}{4\lambda}; \\ \lambda &= \frac{1}{2\beta_A^2} + \frac{1}{2\beta_B^2} + \frac{\varepsilon_1^2}{2\beta_{dA}^2} + \frac{\varepsilon_3^2}{2\beta_{dB}^2}; \\ \mu &= \frac{\varepsilon_1}{\beta_{dA}^2} + \frac{\varepsilon_3}{\beta_{dB}^2}; \quad \nu = \frac{\varepsilon_1}{\beta_{dA}^2} + \frac{\varepsilon_3}{\beta_B^2} + \frac{\varepsilon_3^2}{\beta_{dB}^2}; \\ \varepsilon_1 &= \frac{m_1}{m_1+m_2}; \quad \varepsilon_2 = \frac{m_3}{m_1+m_3+m_5}; \\ \varepsilon_3 &= \frac{m_5}{m_2+m_5}; \quad \varepsilon_4 = \frac{m_4}{m_1+m_4},\end{aligned}$$

TABLE XIII: The flavor matrix element ξ for different decay channels of the charmed baryons.

Initial state	Final states				
Λ_c^+	$\Sigma_c^{++}, +, 0 \pi^{-}, 0, +$	$D^+ n / D^0 p$			
	$\sqrt{1/3}$	$\sqrt{1/3}$			
Σ_c^{++}	$\Sigma_c^{++}, + \pi^{0, +}$	$\Lambda_c^+ \pi^+$	$D^+ p$		
	$\sqrt{1/3}$	$\sqrt{1/3}$	$\sqrt{1/6}$		
Σ_c^+	$\Sigma_c^{++}, 0 \pi^{-}, +$	$\Lambda_c^+ \pi^0$	$D^+ n / D^0 p$		
	$\sqrt{1/3}$	$-\sqrt{1/3}$	$\sqrt{1/12}$		
Σ_c^0	$\Sigma_c^+ \pi^- / \Sigma_c^0 \pi^0$	$\Lambda_c^+ \pi^-$	$D^0 n$		
	$\sqrt{1/3}$	$\sqrt{1/3}$	$\sqrt{1/6}$		
$\Xi_c^{(\prime)+}$	$\Lambda_c^+ K^0$	$\Xi_c^{(\prime)+} \pi^+$	$\Xi_c^{(\prime)+} \pi^0$	$\Sigma_c^{++} K^-$	$\Sigma_c^+ K^0$
	$\sqrt{1/6}$	$\sqrt{1/6}$	$\sqrt{1/12}$	$\sqrt{1/3}$	$\sqrt{1/6}$
$\Xi_c^{(\prime)0}$	$\Lambda_c^+ K^-$	$\Xi_c^{(\prime)+} \pi^-$	$\Xi_c^{(\prime)0} \pi^0$	$\Sigma_c^+ K^-$	$\Sigma_c^0 K^0$
	$\sqrt{1/6}$	$\sqrt{1/6}$	$\sqrt{1/12}$	$\sqrt{1/6}$	$\sqrt{1/3}$

and

$$q = \frac{\sqrt{[M_A^2 - (M_B + M_C)^2][M_A^2 - (M_B - M_C)^2]}}{2M_A}. \quad (\text{A10})$$

Here, M_A , M_B , and M_C are the masses of hadrons A , B , and C , respectively. Then the $\tilde{\beta}$ in Eq. (16) is given by

$$\tilde{\beta} = \sqrt{\frac{2f}{4fh - g^2}}. \quad (\text{A11})$$

Since for the decay channel of $\Sigma_c(2520) \rightarrow \Lambda_c(2280)\pi$, the value of $C_{jC, j_A, J_A}^{s_Q, j_B, J_B}$ is -1 and we obtain

$$\mathcal{M}_{1\frac{1}{2}}^{1,0}(q) = \frac{3g}{8\pi^{5/4}f^{5/2}\lambda^{3/2}\beta_A^{3/2}\beta_{dA}^{3/2}\beta_B^{3/2}\beta_{dB}^{3/2}\beta_C^{3/2}}, \quad (\text{A12})$$

where a phase space factor $(2\pi E_B E_C / M_A)^{1/2} \gamma$ is omitted. One notices that the unitary rotation between the $L-S$ coupling and $j-j$ coupling (Eq. (11)) should be performed to reduce the transition factors of $1P$ state with the same J^P . More details for calculating the decay amplitudes of an excited baryon in the 3P_0 model can be found in the Refs. [24, 78].

Appendix B: Flavor factors

Based on the light $SU(3)$ flavor symmetry, the flavor wave functions of charmed and charmed-strange baryons are given by

$$\begin{aligned}\Lambda_c^+ &= \frac{1}{\sqrt{2}}(ud - du)c; & \Sigma_c^{++} &= uuc; \\ \Xi_c^+ &= \frac{1}{\sqrt{2}}(us - su)c; & \Sigma_c^+ &= \frac{1}{\sqrt{2}}(ud + du)c; \\ \Xi_c^0 &= \frac{1}{\sqrt{2}}(ds - sd)c; & \Sigma_c^0 &= ddc; \\ \Xi_c^{\prime+} &= \frac{1}{\sqrt{2}}(us + su)c; & \Xi_c^{\prime0} &= \frac{1}{\sqrt{2}}(ds + sd)c.\end{aligned}$$

As shown in Fig.3, the final states of an excited charmed baryons may contain a light meson and a low energy charmed baryon or a light baryon and a charmed meson. The flavor wave functions for the final states are collected in the following

$$\begin{aligned}\pi^+ &= u\bar{d}; & \pi^- &= d\bar{u}; & \pi^0 &= (u\bar{u} - d\bar{d})/\sqrt{2}; \\ K^- &= \bar{u}s; & \bar{K}^0 &= \bar{d}s; & D^+ &= \bar{d}c; & D^0 &= \bar{u}c; \\ p &= \frac{1}{\sqrt{2}}(du - ud)u; & n &= \frac{1}{\sqrt{2}}(du - ud)d; \\ \Lambda^0 &= \frac{1}{\sqrt{2}}(du - ud)s.\end{aligned}$$

With the above flavor wave functions, the flavor matrix elements ξ for different decay processes are presented in Table XIII.

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